

Plasmon and Screening Effects on the Occurrence Scattering Time Advance in Hot Quantum Plasmas

Sang-Chul Na^a and Young-Dae Jung^{a,b}

^a Department of Applied Physics, Hanyang University, Ansan, Kyunggi-Do 426-791, South Korea

^b Atomic and Molecular Data Research Center, National Institute for Fusion Science, Toki, Gifu, 509-5292, Japan

Reprint requests to Prof. Y.-D. J.; E-mail: ydjung@hanyang.ac.kr

Z. Naturforsch. **63a**, 791 – 796 (2008); received June 3, 2008

The plasmon and screening effects on the occurrence scattering time advance for elastic electron-ion collisions are investigated in hot quantum plasmas. The effective screening model taking into account the plasmon and screening effects is employed to represent the electron-ion interactions in hot quantum plasmas. The Glauber method is applied to obtain the occurrence scattering time as a function of the scattering angle, impact parameter, plasmon parameter, collision energy, and Debye length. It is shown that the occurrence scattering time advance is enhanced with increasing the plasmon effect and scattering angle. In addition, it is shown that the occurrence scattering time advance decreases with increasing the collision energy. It is also found that the plasmon effect is dominant for the forward scattering domain and, however, the screening effect is getting important with an increase of the scattering angle.

Key word: Occurrence Scattering Time.

It has been shown that the time-dependence mechanism [1, 2] reveals significant aspects of scattering processes in atomic physics. Moreover, the occurrence scattering time advance [3, 4] for the elastic electron-ion interaction, characterizing the quantum collision process, represents the time of emergence of a moving wave packet during collision. In addition, the angular dependence of the occurrence scattering time for the elastic collision has provided useful information on the scattering mechanism in many areas of physics. The electron-ion collision process in plasmas [5–7] has been of great interest since this process can be used for plasma diagnostics and the description of a many-particle correlation is one of the most interesting problems in plasma physics. It has been also shown that a plasma described by the conventional Debye-Hückel model is classified as a classical ideal plasma since the average interaction energy between charged particles is known to be smaller than the average kinetic energy of the particle in the plasma [8]. Hence, it is obvious that the correlation effects caused by simultaneous interaction of many charged particles in plasmas have to be taken into account together with an increase of the plasma density and temperature in order to properly describe the screened interaction potential in hot quantum plasmas. In these hot quantum plas-

mas, the interaction potential would not be represented by the conventional classical Debye-Hückel model obtained by the Maxwell-Boltzmann velocity distribution of charged particles because of the plasmon effects caused by collective plasma oscillations [9]. Hence, the occurrence scattering time advance for an elastic electron-ion collision in hot quantum plasmas would be quite different from that in classical weakly coupled plasmas. However, the plasmon and screening effects on the occurrence scattering time advance have not been investigated yet. Thus, in the present paper we will investigate the plasmon and screening effects on the occurrence scattering time advance for an elastic electron-ion collision in hot quantum plasmas. The effective screening potential taking into account the plasmon and screening effects is employed to describe the electron-ion interactions in hot quantum plasmas. The Glauber method [10] is applied to obtain the complex scattering amplitude and occurrence scattering time advance as functions of the plasma parameters of hot quantum plasmas.

In the presence of the interaction potential $V(\mathbf{r})$, the space wave function $\Psi_k(\mathbf{r})$ satisfies the Schrödinger equation

$$(\nabla^2 + k^2)\Psi_k(\mathbf{r}) = \frac{2\mu}{\hbar^2}V(\mathbf{r})\Psi_k(\mathbf{r}), \quad (1)$$

where \mathbf{r} is the position vector, $k [= (2\mu E/\hbar^2)^{1/2}]$ is the wave number, μ is the reduced mass of the collision system, $E (= \mu v^2/2)$ is the collision energy, and v is the collision velocity. It is indispensable to investigate the scattering amplitude in order to obtain the information on the scattering mechanism of the system since the scattering process would be reduced to the problem of finding the scattering amplitude. Using the Glauber method [10] for the potential scattering with the wave function $\Psi_k(\mathbf{r})$ and free outgoing Green's function of the Helmholtz operator $(\nabla^2 + k^2)$, $G^{(+)}(\mathbf{r}, \mathbf{r}') (= -e^{-ik|\mathbf{r}-\mathbf{r}'|}/4\pi|\mathbf{r}-\mathbf{r}'|)$ [2], in the cylindrical coordinates such as $\mathbf{r} = \mathbf{b} + z\hat{\mathbf{n}}$, where \mathbf{b} is the impact parameter, $\hat{\mathbf{n}}$ is the unit vector perpendicular to the momentum transfer $\mathbf{q} (\equiv \mathbf{k}_i - \mathbf{k}_f)$, \mathbf{k}_i and \mathbf{k}_f are, respectively, the incident and final wave vectors, the complex scattering amplitude [2], $f(\mathbf{k}_i, \mathbf{k}_f)$, can be represented by

$$f(\mathbf{k}_i, \mathbf{k}_f) = -ik_i \int_0^\infty db b J_0(qb) [\exp(i\chi(k_i, b)) - 1] \quad (2)$$

$$= |f(\mathbf{k}_i, \mathbf{k}_f)| \exp[i\eta(\mathbf{k}_i, \mathbf{k}_f)],$$

where $J_0(qb)$ is the zeroth-order Bessel function [11], $\chi(k_i, b) [= \chi_1(b)/k_i]$ is the phase shift [2], $\chi_1(b)$ is given by

$$\chi_1(b) = -\frac{\mu}{\hbar^2} \int_{-\infty}^\infty dz V(r), \quad (3)$$

and $\eta(\mathbf{k}_i, \mathbf{k}_f)$ is the argument of the complex scattering amplitude. For the potential scattering, it is shown that the Glauber method [10] is equivalent to the eikonal method, and the validity is known for $|V|R/\hbar v < 1$, where $|V|$ is the typical strength of the interaction potential and R is the potential action range. If we assume that the centre of the moving wave packet would reach the location of the target ion at the origin at $t = 0$, the occurrence scattering time τ [3, 4] would be represented by the derivative of the argument $\eta(\mathbf{k}_i, \mathbf{k}_f)$ of the complex scattering amplitude with respect to the wave number k_i :

$$\tau(k, \theta) = \frac{\mu}{\hbar k} \left[\frac{\partial \eta(\mathbf{k}_i, \mathbf{k}_f)}{\partial k_i} \right]_{k_i=k_f=k}, \quad (4)$$

where θ is the scattering angle between the wave vectors \mathbf{k}_i and \mathbf{k}_f , and the condition $k_i = k_f = k$ is the requirement of the elastic collision.

The useful analytical expression of the modified interaction potential [9, 12] in hot quantum plasmas has been obtained by the quantum approach including the screening and plasmon effects caused by plasma oscillations. These quantum effects may complicate the depiction of the ordinary Debye-Hückel interaction between charged particles in quantum plasmas. Using this modified interaction model [9], the effective interaction potential $V_{\text{eff}}(r, \beta, L)$ between the electron and ion with charge Ze in hot quantum plasmas is given by

$$V_{\text{eff}}(r, \beta, L) = -\frac{Ze^2}{r} \frac{1}{4(1-\beta^2)^{1/2}} \cdot \left[(4-\beta)e^{-r/L_1} - 2(1-(1-\beta^2)^{1/2})e^{-r/L_2} \right], \quad (5)$$

where $\beta (\equiv \hbar\omega_0/E_T)$ is the plasmon parameter, $\hbar\omega_0$ is the plasmon energy, ω_0 is the plasma frequency, $E_T (= k_B T)$ is the thermal energy, k_B is the Boltzmann constant, T is the plasma temperature, L is the Debye length, L_1 and L_2 are the effective screening lengths, respectively, $L_1 \equiv [1 + (1-\beta^2)^{1/2}]^{1/2}(L/2^{1/2})$ and $L_2 \equiv [1 - (1-\beta^2)^{1/2}]^{1/2}(L/2^{1/2})$. This effective interaction potential is known to be valid for the range of $0 \leq \beta < 1$ since the plasmon energy is expected to be smaller than the thermal energy [9]. If the plasmon effects are neglected in these hot quantum plasmas, the effective interaction potential goes over into the case of the conventional Debye-Hückel (DH) potential $V_{\text{eff}} \rightarrow V_{\text{DH}} = (-Ze^2/r)e^{-r/L}$ since $L_1 \rightarrow L$ and $L_2 \rightarrow 0$ as $\beta \rightarrow 0$. After some mathematical manipulations using the effective potential (5) and identity of the MacDonald function of order n [11]: $K_n(x) [= (\pi^{1/2}/(n-1/2)!) (x/2)^n \int_1^\infty dp e^{-px} (p^2-1)^{n-1/2}]$, the scaled phase shift $\bar{\chi}_1(\bar{b}) [= \bar{k}\chi(\bar{k}, \bar{b})/2]$ of the complex scattering amplitude is obtained as

$$\bar{\chi}_1(\bar{b}, \beta, \bar{L}) = \frac{1}{4(1-\beta^2)^{1/2}} \cdot [(4-\beta)K_0(\bar{b}/\bar{L}_1) - 2(1-(1-\beta^2)^{1/2})K_0(\bar{b}/\bar{L}_2)], \quad (6)$$

where $\bar{k} (= ka_Z)$ is the scaled wave number, $a_Z (= a_0/Z)$ is the Bohr radius of the hydrogenic ion with nuclear charge Ze , $a_0 (= \hbar^2/me^2)$ is the Bohr radius of the hydrogen atom, m is the electron mass, $\bar{b} (\equiv b/a_Z)$ is the scaled impact parameter, and $\bar{L} (\equiv L/a_Z)$ is the scaled Debye length, \bar{L}_1 and \bar{L}_2 are the scaled screening lengths, respectively, $\bar{L}_1 (\equiv L_1/a_Z) = [1 + (1-\beta^2)^{1/2}]^{1/2}(\bar{L}/2^{1/2})$ and $\bar{L}_2 (\equiv L_2/a_Z) = [1 - (1-\beta^2)^{1/2}]^{1/2}(\bar{L}/2^{1/2})$. In high collision energies, the

complex scattering amplitude would be expanded as a power series in the potential strength in the following form:

$$f(\bar{k}_i, \bar{k}_f, \theta) = f_R(\bar{k}_i, \bar{k}_f, \theta) + i f_I(\bar{k}_i, \bar{k}_f, \theta), \quad (7)$$

where $\bar{k}_i \equiv k_i a_Z$, $\bar{k}_f \equiv k_f a_Z$, $f_R(\bar{k}_i, \bar{k}_f, \theta)$ and $f_I(\bar{k}_i, \bar{k}_f, \theta)$ are the real and imaginary parts of the complex scattering amplitude, respectively,

$$f_R(\bar{k}_i, \bar{k}_f, \theta) = 2a_Z \int_0^\infty d\bar{b} \bar{b} J_0(\bar{q} \bar{b}) \bar{\chi}_1(\bar{b}, \beta, \bar{L}), \quad (8)$$

$$f_I(\bar{k}_i, \bar{k}_f, \theta) = \frac{2a_Z}{\bar{k}_i} \int_0^\infty d\bar{b} \bar{b} J_0(\bar{q} \bar{b}) \bar{\chi}_1^2(\bar{b}, \beta, \bar{L}), \quad (9)$$

and $\bar{q}(\bar{k}_i, \bar{k}_f, \theta) [\equiv qa_Z = (\bar{k}_i^2 + \bar{k}_f^2 - 2\bar{k}_i \bar{k}_f \cos \theta)^{1/2}]$ is the scaled momentum transfer. Here, the imaginary part of the forward scattering amplitude, $f_I(\bar{k}_i, \bar{k}_f, \theta = 0)$, leads us the useful information on the total elastic electron-ion collision cross-section according to the optical theorem [13]. The occurrence scattering time advance for the elastic electron-ion collision in hot quantum plasmas including the plasmon and screening effects is then obtained as

$$\begin{aligned} \bar{\tau}(\bar{E}, \beta, \bar{L}, \theta) &\equiv \tau(\bar{E}, \beta, \bar{L}, \theta) v / a_Z \\ &= \left[f_R(\bar{k}_i, \bar{k}_f, \theta) \frac{\partial f_I(\bar{k}_i, \bar{k}_f, \theta)}{\partial \bar{k}_i} \right. \\ &\quad \left. - f_I(\bar{k}_i, \bar{k}_f, \theta) \frac{\partial f_R(\bar{k}_i, \bar{k}_f, \theta)}{\partial \bar{k}_i} \right] \Big|_{\bar{k}_i = \bar{k}_f = \bar{E}^{1/2}} \\ &\quad / [f_R^2(\bar{k}_i, \bar{k}_f, \theta) + f_I^2(\bar{k}_i, \bar{k}_f, \theta)] \Big|_{\bar{k}_i = \bar{k}_f = \bar{E}^{1/2}}, \end{aligned} \quad (10)$$

where $\bar{E} (= \bar{k}^2)$ is the scaled collision energy. The real part of the scattering amplitude $f_R(\bar{k}_i, \bar{k}_f, \theta) \Big|_{\bar{k}_i = \bar{k}_f = \bar{E}^{1/2}}$ is given by

$$\begin{aligned} f_R(\bar{k}_i, \bar{k}_f, \theta) \Big|_{\bar{k}_i = \bar{k}_f = \bar{E}^{1/2}} &= \frac{a_Z}{2(1 - \beta^2)^{1/2}} \int_0^\infty d\bar{b} \bar{b} \\ &\cdot J_0(2\bar{E}^{1/2} \bar{b} \sin(\theta/2)) [(4 - \beta) K_0(2^{1/2}(1 + \\ &(1 - \beta^2)^{1/2})^{-1/2} \bar{b} / \bar{L}) - 2(1 - (1 - \beta^2)^{1/2}) \\ &\cdot K_0(2^{1/2}(1 - (1 - \beta^2)^{1/2})^{-1/2} \bar{b} / \bar{L})]. \end{aligned} \quad (11)$$

The imaginary part of the scattering amplitude $f_I(\bar{k}_i, \bar{k}_f, \theta) \Big|_{\bar{k}_i = \bar{k}_f = \bar{E}^{1/2}}$ can be represented by

$$f_I(\bar{k}_i, \bar{k}_f, \theta) \Big|_{\bar{k}_i = \bar{k}_f = \bar{E}^{1/2}} = \frac{a_Z}{8\bar{E}^{1/2}(1 - \beta^2)}$$

$$\begin{aligned} &\cdot \int_0^\infty d\bar{b} \bar{b} J_0(2\bar{E}^{1/2} \bar{b} \sin(\theta/2)) \\ &\cdot [(4 - \beta) K_0(2^{1/2}(1 + (1 - \beta^2)^{1/2})^{-1/2} \bar{b} / \bar{L}) \\ &- 2(1 - (1 - \beta^2)^{1/2}) \\ &\cdot K_0(2^{1/2}(1 - (1 - \beta^2)^{1/2})^{-1/2} \bar{b} / \bar{L})]^2. \end{aligned} \quad (12)$$

The derivative of the real part of the scattering amplitude $\partial f_R / \partial \bar{k}_i \Big|_{\bar{k}_i = \bar{k}_f = \bar{E}^{1/2}}$ with respect to the scaled wave number \bar{k}_i is expressed as

$$\begin{aligned} \frac{\partial f_R(\bar{k}_i, \bar{k}_f, \theta)}{\partial \bar{k}_i} \Big|_{\bar{k}_i = \bar{k}_f = \bar{E}^{1/2}} &= -\frac{a_Z}{2(1 - \beta^2)^{1/2}} \sin(\theta/2) \\ &\cdot \int_0^\infty d\bar{b} \bar{b}^2 J_1(2\bar{E}^{1/2} \bar{b} \sin(\theta/2)) \\ &\cdot [(4 - \beta) K_0(2^{1/2}(1 + (1 - \beta^2)^{1/2})^{-1/2} \bar{b} / \bar{L}) \\ &- 2(1 - (1 - \beta^2)^{1/2}) \\ &\cdot K_0(2^{1/2}(1 - (1 - \beta^2)^{1/2})^{-1/2} \bar{b} / \bar{L})]. \end{aligned} \quad (13)$$

Finally the derivative of the imaginary part of the scattering amplitude $\partial f_I / \partial \bar{k}_i \Big|_{\bar{k}_i = \bar{k}_f = \bar{E}^{1/2}}$ with respect to the scaled wave number \bar{k}_i is expressed by two integrals in the form

$$\begin{aligned} \frac{\partial f_I(\bar{k}_i, \bar{k}_f, \theta)}{\partial \bar{k}_i} \Big|_{\bar{k}_i = \bar{k}_f = \bar{E}^{1/2}} &= -\frac{a_Z}{8\bar{E}^{1/2}(1 - \beta^2)} \sin(\theta/2) \\ &\cdot \int_0^\infty d\bar{b} \bar{b}^2 J_1(2\bar{E}^{1/2} \bar{b} \sin(\theta/2)) \\ &\cdot [(4 - \beta) K_0(2^{1/2}(1 + (1 - \beta^2)^{1/2})^{-1/2} \bar{b} / \bar{L}) - 2(1 - \\ &(1 - \beta^2)^{1/2}) K_0(2^{1/2}(1 - (1 - \beta^2)^{1/2})^{-1/2} \bar{b} / \bar{L})]^2 \\ &- \frac{a_Z}{8\bar{E}(1 - \beta^2)} \int_0^\infty d\bar{b} \bar{b}^2 J_0(2\bar{E}^{1/2} \bar{b} \sin(\theta/2)) \\ &\cdot [(4 - \beta) K_0(2^{1/2}(1 + (1 - \beta^2)^{1/2})^{-1/2} \bar{b} / \bar{L}) \\ &- 2(1 - (1 - \beta^2)^{1/2}) \\ &\cdot K_0(2^{1/2}(1 - (1 - \beta^2)^{1/2})^{-1/2} \bar{b} / \bar{L})]^2. \end{aligned} \quad (14)$$

The dynamic screening effects are neglected in this work since we are interested in the occurrence scattering time advance in hot plasmas. However, for collision and radiation processes in dense cold plasmas, the dynamical screened interactions have to be considered. The detailed discussions on the dynamical

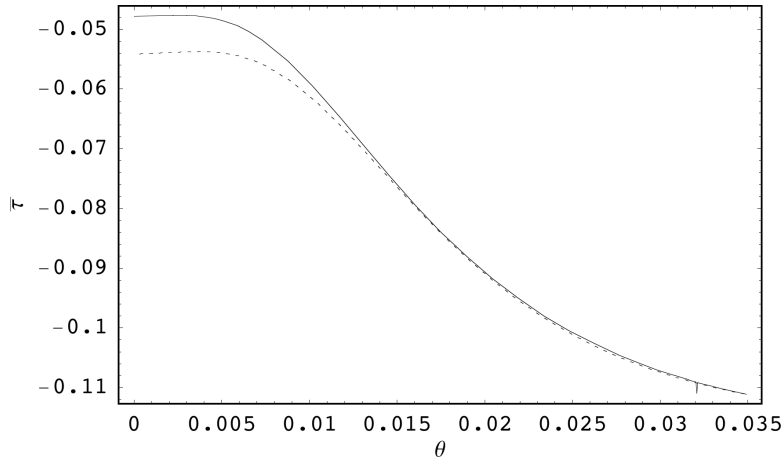


Fig. 1. The scaled occurrence scattering time advance ($\bar{\tau}$) as a function of the scattering angle (θ) in units of radians for $\bar{E} = 10$ and $\bar{L} = 50$. The solid line represents the case of $\beta = 0.1$. The dotted line represents the case of $\beta = 0.8$.

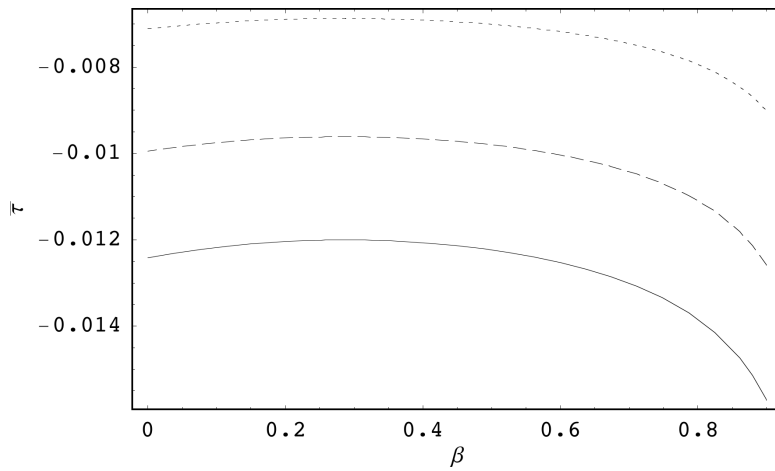


Fig. 2. The scaled occurrence scattering time advance ($\bar{\tau}$) for the forward scattering case as a function of the plasmon parameter (β) for $\bar{L} = 50$. The solid line represents the case of $\bar{E} = 40$. The dashed line represents the case of $\bar{E} = 50$. The dotted line represents the case of $\bar{E} = 70$.

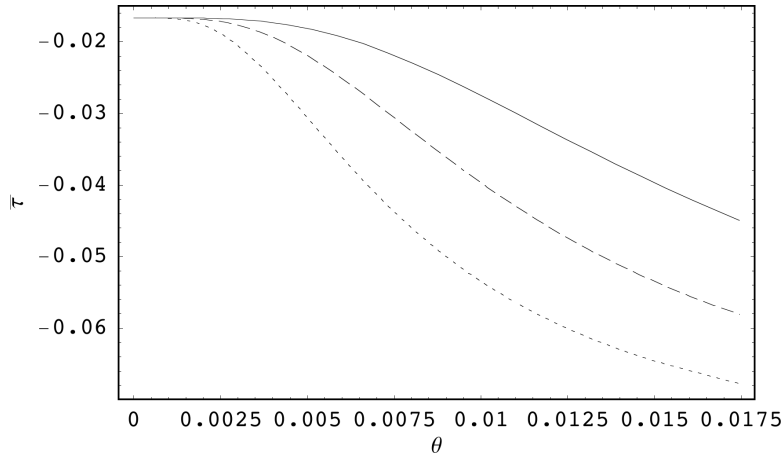


Fig. 3. The scaled occurrence scattering time advance ($\bar{\tau}$) as a function of the scattering angle (θ) in units of radians for $\beta = 0.6$ and $\bar{E} = 30$. The solid line represents the case of $\bar{L} = 40$. The dashed line represents the case of $\bar{L} = 60$. The dotted line represents the case of $\bar{L} = 100$.

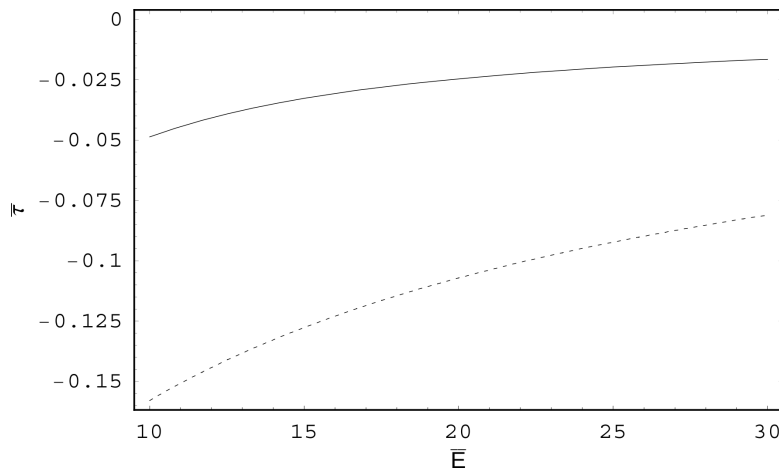


Fig. 4. The scaled occurrence scattering time advance ($\bar{\tau}$) as a function of the scaled collision energy (\bar{E}) for the forward scattering direction. The solid line is the result for $n \approx 10^{31} \text{ m}^3$ and $T \approx 10^4 \text{ eV}$ (inertial confinement fusion plasmas) and the dotted line for $n \approx 10^{20} \text{ m}^3$ and $T \approx 10^4 \text{ eV}$ (magnetic confinement fusion plasmas).

screening process based on a Green function approach to treat dispersive and frequency-dependent interactions can be found in the excellent monograph by Kremp, Schlages, and Kraeft [15].

In order to specifically investigate the plasmon and screening effects on the occurrence scattering time advance for the elastic electron-ion collision in hot quantum plasmas, we consider $\bar{E} > 1$ since the Glauber or eikonal method [10] is known to be valid for high collision energies. Figure 1 shows the scaled occurrence scattering time advance ($\bar{\tau}$) as a function of the scattering angle (θ) for two values of the plasmon parameter (β). From this figure, we find that the occurrence scattering time advance increases with an increase of the scattering angle. It is also shown that the plasmon effects decrease with increasing the scattering angle. Thus, it is found that the plasmon effects are more significant for the case of forward scattering. Figure 2 presents the scaled occurrence scattering time advance ($\bar{\tau}$) for the forward scattering case, i. e., $\theta = 0$, as a function of the plasmon parameter (β) for various values of the collision energy (\bar{E}). As it is seen, it is found that the occurrence scattering time advance increases with increasing the plasmon parameter. Hence, the quantum plasmon effect enhances the occurrence scattering time advance for the electron-ion scattering in hot quantum plasmas. In addition, it is shown that the occurrence scattering time advance decreases with increasing the collision energy. Figure 3 presents the scaled occurrence scattering time advance ($\bar{\tau}$) as a function of the scattering angle (θ) for various values of the Debye length (\bar{L}). Figure 4 presents the scaled occurrence scattering time advance ($\bar{\tau}$) as a function

of the scaled collision energy (\bar{E}) for inertial confinement fusion plasmas ($n \approx 10^{31} \text{ m}^3$, $T \approx 10^4 \text{ eV}$) and magnetic confinement fusion plasmas ($n \approx 10^{20} \text{ m}^3$, $T \approx 10^4 \text{ eV}$). As it is shown, the occurrence scattering time advance is found to increase with an increase of the Debye length. Thus, the plasma screening effect suppresses the occurrence scattering time advance for an elastic electron-ion collision in plasmas. In addition, it is shown that the plasma screening effect vanishes for the forward scattering case. From Figs. 1 and 4, we also find that the plasmon effect is dominant for the forward scattering domain and, however, the screening effect is getting important with an increase of the scattering angle. Therefore, the plasmon and screening effects play significant roles in scattering processes in hot quantum plasmas. Since the occurrence time delay or advance is expected to be revealed for a given scattering angle, the occurrence scattering time is related to the electric conductivity in plasmas. This paper shows the possibility of detection of the occurrence scattering time as a function of the scattering angle. Hence, in the future, we will detect and resolve the plasma screening effects on the angular dependence of the occurrence scattering time in various plasmas. These results will provide useful information of the quantum and plasma screening effects on the occurrence scattering time phenomenon due to the elastic collisions in quantum plasmas.

Acknowledgement

Y.-D. J. gratefully acknowledges the Director-General Professor O. Motojima and Professor D. Kato for warm hospitality while visiting the National In-

stitute for Fusion Science (NIFS) in Japan. The authors would like to thank the anonymous referees for suggesting improvements to this text. This work was

supported by the Korea Research Foundation Grant funded by the Korean Government (MOEHRD, Basic Research Promotion Fund) (KRF-2007-313-C00169).

- [1] U. Fano and A. R. P. Rau, *Atomic Collisions and Spectra*, Academic Press, Orlando 1986.
- [2] T. Kopaleishvili, *Collision Theory*, World Scientific, Singapore 1995.
- [3] T. Suzuki, *Europhys. Lett.* **9**, 513 (1989).
- [4] T. Suzuki, *Prog. Theor. Phys.* **98**, 271 (1997).
- [5] V. P. Shevelko and L. A. Vainshtein, *Atomic Physics for Hot Plasmas*, Institute of Physics, Bristol 1993.
- [6] V. P. Shevelko, *Atoms and their Spectroscopic Properties*, Springer, Berlin 1997.
- [7] V. P. Shevelko and H. Tawara, *Atomic Multielectron Processes*, Springer, Berlin 1998.
- [8] D. Zubarev, V. Morozov, and G. Röpke, *Statistical Mechanics of Nonequilibrium Processes*, Akademie-Verlag, Berlin 1996.
- [9] J. Kvasnica and J. Horáček, *Czech. J. Phys. B* **25**, 325 (1975).
- [10] G. A. Pavlov, *Transport Processes in Plasmas with Strong Coulomb Interaction*, Gordon and Breach, Amsterdam 2000.
- [11] E. T. Whittaker and G. N. Watson, *A Course of Modern Analysis*, 4th ed., Cambridge University Press, Cambridge 1952.
- [12] J. Krcik, B. Sestak, and L. Aubrecht, *Foundations of Classical and Quantum Plasma Physics*, Academia, Prague 1974.
- [13] R. Peierls, *Surprises in Theoretical Physics*, Princeton University Press, Princeton 1979.
- [14] D. Kremp, M. Schlages, and W.-D. Kraeft, *Quantum Statistics of Nonideal Plasmas*, Springer, Berlin 2005.